

## Modelling the liquidity premium on corporate bonds

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### Abstract

The liquidity premium on corporate bonds has been high on the agenda of Solvency regulators owing to its potential relationship to an additional discount factor on long-dated insurance liabilities. We analyse components of the credit spread as a function of standard bond characteristics during 2003–2014 on a daily basis by regression analyses, after introducing a new liquidity proxy. We derive daily distributions of illiquidity contributions to the credit spread at the individual bond level and find that liquidity premia were close to zero just before the financial crisis. We observe the time-varying nature of liquidity premia as well as a widening in the daily distribution in the years after the credit crunch. We find evidence to support higher liquidity premia, on average, on bonds of lower credit quality. The evolution of model parameters is economically intuitive and brings additional insight into investors' behaviour. The frequent and bond-level estimation of liquidity premia, combined with few data restrictions makes the approach suitable for ALM modelling, especially when future work is directed towards arriving at forward-looking estimates at both the aggregate and bond-specific level.

### Keywords

Liquidity premium; Corporate bonds; Credit spread; Bid-ask spread

### JEL classification

G12; G22; G24.

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## 1. Introduction

The liquidity premium on corporate bonds (sometimes also referred to as the *illiquidity* premium) is a much discussed topic with respect to the Solvency II framework and potential allowances for adjusted discount factors on long-dated insurance liabilities. In its first report, CEIOPS (Committee of European Insurance and Occupational Pensions Supervisors) stated that “to determine the part of the spread attributable to liquidity risk, the challenge that has to be faced is the accurate breakdown of this spread into its components” (CEIOPS, 2010).

Academic literature has studied the effect of illiquidity on corporate bond prices extensively over the past three decades, from both theoretical and empirical perspectives. Amihud *et al.* (2006) discuss a series of asset pricing models in which frictional costs lead to higher expected returns, compensating investors for investing in illiquid assets. The work in Amihud *et al.* (2006) is a special case of Amihud & Mendelson (1986) where investors have exogenous time horizons and assets bear illiquidity owing to exogenous trading costs. Amihud *et al.* (2006) and Acerbi & Scandolo (2008) also discuss the heterogeneity of corporate bond investors with respect to expected holding times and how these different groups lead to a market equilibrium in which investors with short expected holding periods hold very liquid assets and investors with the longest expected holding periods hold illiquid assets.

A related concept in asset pricing theory is that of the marginal investor that ultimately determines an asset’s price (Sharpe, 1964; Cochrane, 2005). With respect to the corporate bond market, this raises the question whether there are sufficient hold-to-maturity investors to take up the entire supply of corporate bonds; if sufficient long-term investors are vested in the market the yield spreads would only reflect credit factors and liquidity premia would be very small.

Empirical literature investigates whether illiquidity is priced by relating bond prices to various proxies for liquidity using a reduced-form modelling approach, but also aims to quantify the liquidity premium as part of the credit spread. In addition to using reduced-form models to quantify the liquidity premium, structural models of default (e.g. Merton, 1974; Leland & Toft, 1996) have been used, as have direct computation methods (e.g. Breger & Stovel, 2004; Koziol & Sauerbier, 2007). Section 2 discusses the empirical literature in more detail.

In this article, we define the liquidity premium as being the difference in yield to maturity of a bond relative to the yield on a hypothetical perfectly liquid bond with otherwise identical characteristics. For an investor who is prepared to buy and hold to maturity, the liquidity premium represents the expected reward, per annum, in return for sacrificing the option to sell a bond before maturity. Any investor who plans to or might need to sell before maturity will, on average, earn a lower premium than our estimate.

We develop a new methodology for estimating liquidity premia on corporate bonds, addressing some of the pitfalls of other modelling approaches. Using quoted bid-ask prices and a comprehensive data set (Oct 2003–May 2014) of end-of-day bond characteristics and statistics (GBP investment grade), we derive a liquidity measure uncorrelated with bid-ask spreads and bond characteristics. We use the new liquidity measure to extract liquidity premia, but also note that the liquidity score on individual bonds can be a useful tool in ALM modelling or portfolio management.

In addition to deriving a new liquidity measure, our paper estimates liquidity premia on a more granular level than existing literature. Daily cross-sectional regression analysis allows estimation of

liquidity premia at the individual bond level, on a daily basis; again, this can be particularly useful for ALM modelling. Lastly, our paper is novel in the sense that the same methodology for estimating liquidity premia is applied over a relatively long period of time (11 years), capturing both the benign economic climate before the financial crisis, the financial crisis and more recent years. The modelling approach of daily cross-sectional regressions also allows for the evolution of model parameters to be studied. In particular, the time-varying nature of liquidity premia, both in basis points (bps) and in proportion of total credit spread, is clearly visible.

The structure of the paper is as follows. Section 2 will review literature related to liquidity proxies (section 2.1) and liquidity premium estimation methods (section 2.2). Section 3 briefly introduces the iBoxx data set and section 4 details our modelling approach, both from a conceptual (section 4.1) and empirical (section 4.2) perspective. Section 5 discusses the numerical results; we perform an alternative modelling approach and take a closer look at relative bid-ask spread (RBAS) properties in section 6 and section 7 concludes the paper with observations and discussion.

## 2. Literature

As liquidity in financial markets and liquidity of securities are difficult concepts to define and even more difficult to quantify, various proxies for liquidity have been proposed. These liquidity proxies are unlikely to capture all aspects of liquidity (Kyle, 1985) and are generally (heavily) correlated (Dick-Nielsen *et al.*, 2012). The TRACE database of US corporate bond transactions has greatly contributed to the advances in empirical study of various aspects of liquidity. Section 2.1 discusses a non-exhaustive list of common liquidity measures and section 2.2 briefly outlines methodologies that have been used to quantify the liquidity premium on corporate bonds.

### 2.1. Liquidity proxies

Several measures, or proxies, of illiquidity for corporate bonds have been proposed. A simple intuitive measure of illiquidity, central in our study, is the bid-ask spread, which Edwards *et al.* (2007) considered in detail. The Roll (1984) measure, used in Bao *et al.* (2011) and the Imputed Roundtrip Cost as in Feldhütter (2012) or Friewald *et al.* (2012) are other measures of illiquidity related to transaction costs.

A second class of illiquidity measures describes market depth, one of the market liquidity indicators in Kyle (1985), by assessing the price impact of trades. By far the most frequently used liquidity proxy in this class is the Amihud-measure (Amihud, 2002), which captures the daily price response associated with a one currency unit of trading volume and is defined as the ratio of the daily absolute return to the (dollar) trading volume on that day (used in Amihud, 2002; Dick-Nielsen, 2009; De Jong & Driessen, 2012; Dick-Nielsen *et al.*, 2012). Its widespread use also caused the Amihud-measure to be subjected to further study, criticism and refinement. Theoretical work by Brennan *et al.* (2012), questioning the symmetric microstructure framework suggested by Kyle (1985), finds that equilibrium rates of return are sensitive to changes between seller-initiated trades and returns, but not sensitive to buyer-initiated trades. Whereas the Amihud-measure treats positive and negative returns the same, Brennan *et al.* (2013) decomposes the traditional Amihud-measure into components that correspond to up-days and down-days, hinting towards different liquidity in a down-market than in an up-market (e.g. Brunnermeier & Pedersen, 2009). Brunnermeier & Pedersen (2009) find that for US equity markets, the down-day component of the Amihud-measure is associated with a return premium whereas the up-day component is not significantly priced.

A third class of liquidity proxies can be referred to as trading intensity variables, which frequently cover both measures based on turnover and zero-trading-days (Chen *et al.*, 2007). In addition to bond-specific measures, Dick-Nielsen *et al.* (2012) develop a firm-specific zero-trading-days measure; the number of days in a given time period where none of the bonds were issued by a particular firm trade. At any time, this measure tries to capture the fact that issuers will have bonds of varying maturities outstanding and a shorter waiting time between trades within a firm indicates new information about the firm is relatively more frequent.

In addition to using individual liquidity proxies, various aggregate proxies are used. Kerry (2008) builds an index proxy by averaging nine different proxies for liquidity including six microstructure variables evenly split between various bid-ask spread approximations and price impact (all return-to-volume). Dick-Nielsen *et al.* (2012) provide a comprehensive review of many liquidity proxies, all of which are subjected to principal component analysis to both assess communality between individual proxies and create a “new” aggregate liquidity proxy.

## 2.2. Liquidity premium estimation methods

The literature estimating liquidity premia on corporate bonds is vast, but methodologies can be broadly categorised as one of the following three. First, direct approaches, sometimes referred to as model-free approaches, usually rely on finding two financial instruments (or indices/portfolios) that have the exact same characteristics apart from their liquidity. The liquidity premium is then inferred from the difference in (expected) yields between the two instruments (portfolios). Credit default swaps (CDS) are considered a relatively liquid asset as the number of contracts is not fixed and short selling is easy and cheap compared with the corporate bond market (see Brigo *et al.*, 2011 for a discussion of CDS liquidity). Using arbitrage, Duffie (1999) shows that the spread of a corporate floating-rate note (FRN) over a default-free FRN should be equal to the CDS premium. In reality, the difference between the CDS premium and the spread on the bond has been observed to be negative, implying that other factors contribute to observed bond spread. Longstaff *et al.* (2005) interpret this negative basis as the difference in yield between an illiquid corporate bond (synthetically free of expected defaults and credit risk) and the yield on a liquid credit risk-free bond. The residual yield is then interpreted as a direct quantification of the discounted yield associated with liquidity. The negative-basis approach assumes that CDS premia are a direct measure of credit risk, and the negative basis is solely related to illiquidity; both are strong assumptions (see Credit Suisse Group, 2009; Bongaerts *et al.*, 2011; Arora *et al.*, 2012). This CDS price linkage broke down altogether during the financial crisis. Even if the strong assumptions were to hold and the negative basis is compensation for illiquidity, the approach can be impractical, despite its ease of computation. In addition to methodological issues, many issuers do not have CDS contracts trading (potential selection bias) and a (maturity) mismatch between bond portfolio and reference CDS index is likely.

The second class of models are empirical applications of structural models of default (building on Merton, 1974) that attempt to describe the dynamics between debt structure and probability of default. As a tool to accurately describe the credit risk of a company, the commercial application of KMV (Moody's) is most well known (Bharath & Shumway, 2008). As models explaining the dynamics of the firm, the literature has extended the original Merton model to address some simplistic assumptions made by the original Merton model. Some of the more important extensions can be listed as those relating to the specification of the default barrier (Black & Cox, 1976; Leland, 1994), the process for asset values (Zhou, 2001), allowing for more complex debt structures (Leland, 1994; Leland & Toft, 1996; Fan & Sundaresan, 2000) or stochastic interest rates (Longstaff & Schwartz, 1995).

Structural models allow us to study the dynamics of one (or several) factors and the effect on default probability/credit spread. The inability of structural models to match observed credit spreads has been long documented (Jones *et al.*, 1984). More recently, Eom *et al.* (2004) have taken five structural models, calibrated the required parameters of all models to the same sample of 182 bonds during the period 1986–1997 and compared spread predictions across models with observed historical spreads. Eom *et al.* (2004) conclude that the five structural models cannot accurately price corporate debt, especially on the individual bond level, but note that the difficulties are far from limited to an underestimation of spreads. Eom *et al.* (2004) report Merton (1974) and Geske (1977) to consistently underestimate yield spreads, as previous work indicated (Jones *et al.*, 1984). Longstaff & Schwartz (1995), Leland & Toft (1996) and Collin-Dufresne *et al.* (2001) on average produce yield spreads that are too low. Models by Longstaff & Schwartz (1995) and Collin-Dufresne *et al.* (2001) have a high prediction error on a bond-to-bond basis, that is of a magnitude several times the average prediction error.

An influential empirical application of structural models, used to measure illiquidity premia (residual of observed market spread and estimated fair credit spread) over the period 1997–2007, is by the Bank of England (Webber, 2007). Calibrating the Leland & Toft (1996) model to aggregated UK investment grade corporate bond data, they estimate a liquidity premium of ~50% of credit spread, with small variations over time.

Lastly, reduced-form models, often regression models, use one or several of the liquidity proxies discussed in section 2.1. A non-exhaustive overview of literature would include De Jong & Driessen (2012), Houweling *et al.* (2005), Chen *et al.* (2007), Han & Zhou (2008), Bao *et al.* (2011), Dastidar & Phelps (2011) and Dick-Nielsen *et al.* (2012). All papers use a variety of liquidity proxies, on different bond data sets (by currency, e.g. USD, EUR; and by credit quality, e.g. investment grade or high yield) over different periods of time, and all conclude non-zero, positive liquidity premia.

### 3. Data

We use iBoxx GBP Investment Grade Index data for corporate bonds, for which eligibility for inclusion is based on several selection criteria. The following bond types are specifically excluded: bonds with American call options, FRNs and other fixed to floater bonds, optionally and mandatory convertible bonds, subordinated bank or insurance debt with mandatory contingent conversion features, CDOs or bonds collateralised by CDOs. In addition, retail bonds and private placements are reviewed by the iBoxx Technical Committee on an individual basis and excluded if deemed unsuitable (Markit, 2012a).

All bonds in the Markit iBoxx GBP universe must have a Markit iBoxx Rating of investment grade (Markit, 2012b). The average rating of Fitch Ratings, Moody's Investors Service and Standard & Poor's Rating Services determines the iBoxx rating. Investment grade is defined as BBB – or higher from Fitch and Standard & Poors and Baa3 or higher from Moodys. Ratings from the rating agencies are converted to numerical scores and averaged, then consolidated to the nearest rating grade; the iBoxx Rating system does not use tranches. Eligibility for inclusion is also conditional on the amount outstanding, where the issue needs to be of a minimum size. Gilts need to have an outstanding amount of at least GBP 2 billion, whereas the minimum amount for non-Gilts is set to 250 million.

In our sample (October 2003–July 2014) of 2,767 trading days we observe 2,392 unique bonds from 749 different issuers, with data for ~1,300 bonds on any given day. Our analysis uses a range of analytical values (Markit, 2014) included in the index. These bond characteristics can be contractual (e.g. coupon rate, issuer, maturity, seniority, date of issue, industry) or time dependent (e.g. bid- and ask prices, credit rating, credit spread).

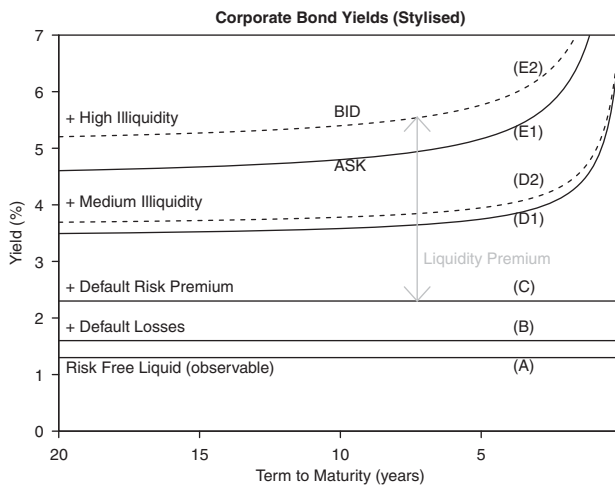
The database reports a number of different measures of the credit spread including annual benchmark spread, option adjusted spread and Z-spread (Markit, 2014). Of these, only the annual benchmark spread is reported for the full duration of the data set and this is used as the measure of the credit spread in our statistical analysis. Our analysis has been repeated over shorter periods with the alternative measures of credit spread and the results have been found to be robust.

Markit iBoxx Index calculations are based on multi-sourced pricing, which, depending on the structure of each market, takes into account a variety of data inputs such as transaction data, quotes from market makers and other observable data points. For the GBP Corporate Index we are using, the source of data is quotes from market makers. Currently, ten market makers submit prices, including Barclays Capital, Goldman Sachs, HSBC, Deutsche Bank and JP Morgan. All submitted prices and quotes have to pass through a three-step consolidation process before being included in the end-of-day value (Markit, 2008).

## 4. Modelling Process

### 4.1. Conceptually

We define the illiquidity premium as the difference in spread between a bond’s observed spread in the market and the spread of a hypothetical bond, identical in all aspects, but perfectly liquid. Figure 1 illustrates this concept further (highly stylised);



**Figure 1.** Stylised representation of bond yields, illustrating the challenge to estimate yield curve C in order to extract liquidity premia. Yield curves: A, risk free (e.g. gilts); B, as A plus expected default losses; C, as B plus credit risk premium; D<sub>1,2</sub> as C plus liquidity premium and bid-ask spread; E<sub>1,2</sub> as D but higher bid-ask spread.

- A represents the yield curve for risk free, perfectly liquid bonds, against which credit spreads are measured.
  - B (not observable) adds in expected default losses for perfectly liquid corporate bonds of a given rating.
  - C (not observable) adds in a risk premium for default losses (sometimes referred to as the allowance for unexpected default losses).
  - D1 and D2 represent the ask and bid yields, respectively, on bonds with medium levels of illiquidity.
  - E1 and E2 represent the ask and bid yields, respectively, on bonds with high levels of illiquidity.
- Markit credit spreads are based on bid prices (Markit, 2008). We define the liquidity premium as the difference between an individual bond's credit spread (e.g. E2) and the credit spread for an equivalent but perfectly liquid bond (curve C). Our challenge is that curve C cannot be observed and needs to be estimated.

## 4.2. Modelling methodology

To extract liquidity premia from corporate bond prices, we follow a three-stage modelling process. In the first stage we model the bid-ask spread and derive a new liquidity proxy, the RBAS. The RBAS is a measure of a bond's illiquidity relative to bonds with identical characteristics (on the same day) and is used in the second stage of the modelling process. In the second stage, credit spread is modelled as a function of bond characteristics, including the bond's RBAS. The third and final stage extracts liquidity premia by computing the difference between a bond's observed spread with the hypothetical spread on a perfectly liquid equivalent bond, estimated by extrapolation.

## 4.3. Modelling the bid-ask spread

In the first stage we model the bid-ask spread using bond characteristics. Separate cross-sectional regression models are fitted to each trading day ( $t$ ), for each rating ( $r$ ). A total of 2,767 days  $\times$  4 ratings means a total of 11,068 regression models are fitted:

$$\text{BAS}(i, r, t) = (\text{Ask price} - \text{Bid price}) / \text{Bid price} \in (0, \infty)$$

$$I_X(r, t) = \text{indicating } X : 0 \text{ or } 1$$

$$\log(\text{BAS}(i, r, t)) = c(r, t)$$

$$+ \beta_{1,\text{FIN}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{FIN}}(i)$$

$$+ \beta_{1,\text{NF}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{NF}}(i)$$

$$+ \beta_2(r, t) \times \log \text{Notional}(i, t)$$

$$+ \beta_3(r, t) \times \text{Coupon}(i, t)$$

$$+ \sum_k \beta_k(r, t) \times I_k(i, t)$$

$$+ \epsilon_{\text{BAS}}(i, t) \quad (\text{residual}) \quad (1)$$

where indicator variables are financial (FIN) or non-financial (NF) issuer, sovereign or non-sovereign issuer (for AAA and AA-rated bonds), senior or subordinate (for A and BBB-rated bonds),

collateralised or not-collateralised, bond age ( $\text{Age} < 1/\text{Age} > 1$ ) and debt tier (lower tier 2 (LT2), for A and BBB-rated bonds).

The inclusion of covariates is based on both economic intuition and previous literature; Houweling *et al.* (2005) for example, examine the use of issue size, duration and bond age as liquidity proxies in their regression models. Parameters are estimated using least squares regression.

Our relative liquidity measure (called the relative bid-ask spread) is defined as

$$\text{RBAS}(i, t) = \exp(\epsilon_{\text{BAS}}(i, t)) \quad (2)$$

By design,  $\log(\text{RBAS})$  is uncorrelated with any covariates included in equation (1), which makes for an attractive property; we can interpret RBAS as the bond's liquidity, independent of any bond characteristics (included as covariates). By design, the distribution of  $\log(\text{RBAS})$ , for a given rating and day, is centred around 0, irrespective of rating, day or economic climate. The variance of the distribution is directly related to the quality of fit of the regression analysis (equation (1)) and determines the variation of observed values for RBAS and ultimately variation in the estimated liquidity premium.

#### 4.4. Modelling the credit spread

Credit spreads are modelled using the same approach; bond characteristics are used to explain variation in credit spreads, cross-sectionally, for each trading day and rating (~11,000 regressions):

$$\begin{aligned} \log(\text{CS}(i, r, t)) = & c(r, t) \\ & + \gamma_{1,\text{FIN}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{FIN}}(i) \\ & + \gamma_{1,\text{NF}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{NF}}(i) \\ & + \gamma_2(r, t) \times \log \text{Notional}(i, t) \\ & + \gamma_3(r, t) \times \text{Coupon}(i, t) \\ & + \gamma_4(r, t) \times \text{RBAS}(i, t) \\ & + \sum_k \gamma_k(r, t) \times I_k(i, t) \\ & + \epsilon_{\text{CS}}(i, t) \quad (\text{residual}) \end{aligned} \quad (3)$$

where indicator variables are identical to equation (1).

Corporate debt is classified into senior and subordinated debt, where subordinated debt is mostly issued by financials, but other corporate issuers might be forced to do so if indentures on earlier issues mandate their status as senior bonds. Subordinated debt can be especially risk sensitive as the bond holders only have claims on an issuer's assets after other bond holders (without the upside potential that shareholders enjoy).

Estimated regression coefficients in equation (3) give an insight into which bond characteristics influence credit spreads cross-sectionally, and how this changes over time. It also allows us to test whether illiquidity is positively priced ( $\gamma_4(r, t) > 0$ ), and whether the price of relative illiquidity varies over time (and by rating).



## 4.5. Creating equivalent, perfectly liquid bonds

As in Figure 1, the liquidity premium is interpreted as additional spread of an illiquid bond over its perfectly liquid equivalent, where the perfectly liquid equivalent is not observable in the market. Using regression equation (3), we formulate a model to estimate the spread of a perfectly liquid equivalent bond by extrapolating RBAS to zero. As RBAS is designed to be uncorrelated with any other covariate in equation (3), we extrapolate to zero, without having to make adjustments to other covariates:

$$\begin{aligned}
 \log(\tilde{C}S_{\text{liq}}(i, r, t)) &= \hat{c}(r, t) \\
 &+ \hat{\gamma}_{1,\text{FIN}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{FIN}}(i) \\
 &+ \hat{\gamma}_{1,\text{NF}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{NF}}(i) \\
 &+ \hat{\gamma}_2(r, t) \times \log \text{Notional}(i, t) \\
 &+ \hat{\gamma}_3(r, t) \times \text{Coupon}(i, t) \\
 &+ \hat{\gamma}_4(r, t) \times 0 \quad (\text{perfectly liquid}) \\
 &+ \sum_k \hat{\gamma}_k(r, t) \times I_k(i, t) \\
 &+ \hat{e}_{\text{CS}}(i, t) \times 0 \quad (\text{no residual}) \quad (4)
 \end{aligned}$$

Then, the liquidity premium ( $i, r, t$ ) is easily derived from both the fitted credit spreads ( $\tilde{C}S(i, r, t)$ ) and the estimated perfectly liquid equivalent credit spreads ( $\tilde{C}S_{\text{liq}}(i, r, t)$ ):

$$\begin{aligned}
 \tilde{L}P_{\text{bps}}(i, r, t) &= \tilde{C}S(i, r, t) - \tilde{C}S_{\text{liq}}(i, r, t) \\
 \tilde{L}P_{\%}(i, r, t) &= \frac{\tilde{C}S(i, r, t) - \tilde{C}S_{\text{liq}}(i, r, t)}{\tilde{C}S(i, r, t)} \quad (5)
 \end{aligned}$$

## 5. Numerical Results

By way of example, we focus much of our discussion on A-rated bonds. For other rating classes we obtain similar results and interpretation. Where appropriate we provide charts to compare results across ratings.

### 5.1. Modelling bid-ask spread

As the model in equation (1) has been fitted over a relatively long period of time, we investigate the robustness of the model parameters,  $\beta_k$ , over time, whereby robustness is defined as stability over short periods of time. Given the significant shock financial markets endured during the credit crunch, we can reasonably expect relationships to (temporarily) change as a result. The evolution of several model parameters is shown in Figures 2 and 3. To aid the interpretation of model parameters in both the bid-ask spread and credit spread models, it is important to note the log transformation on some variables (e.g. duration and notional amount). Whereas for indicator variables the range of possible values are well defined (either 0 or 1), the range of possible values for log-transformed variables is less straightforward; hence, we provide a measure of dispersion for each regression covariate in Table 1.

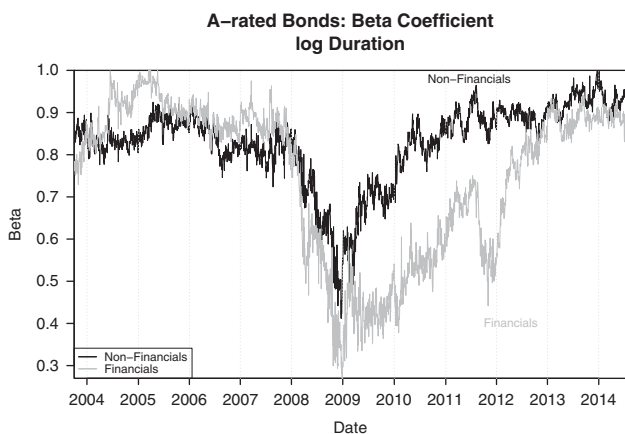


Figure 2. Beta parameters ( $\beta$ ) for log duration, subdivided by financials and non-financials.

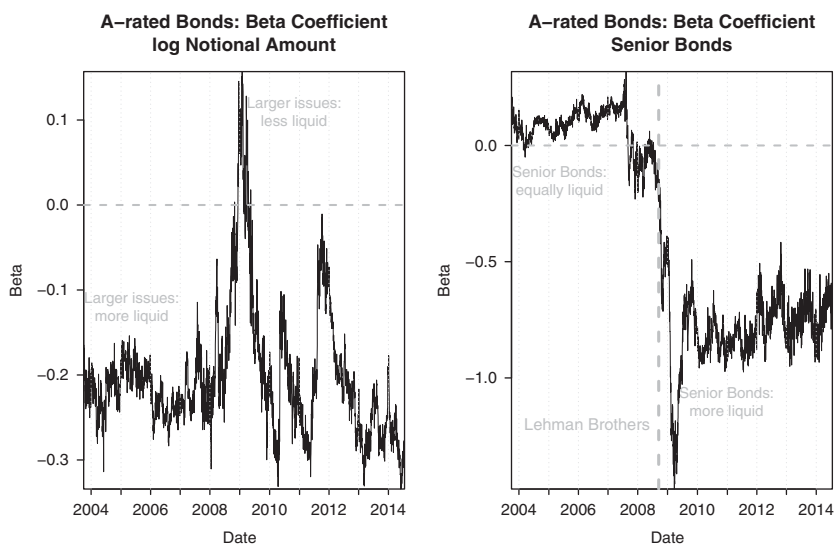
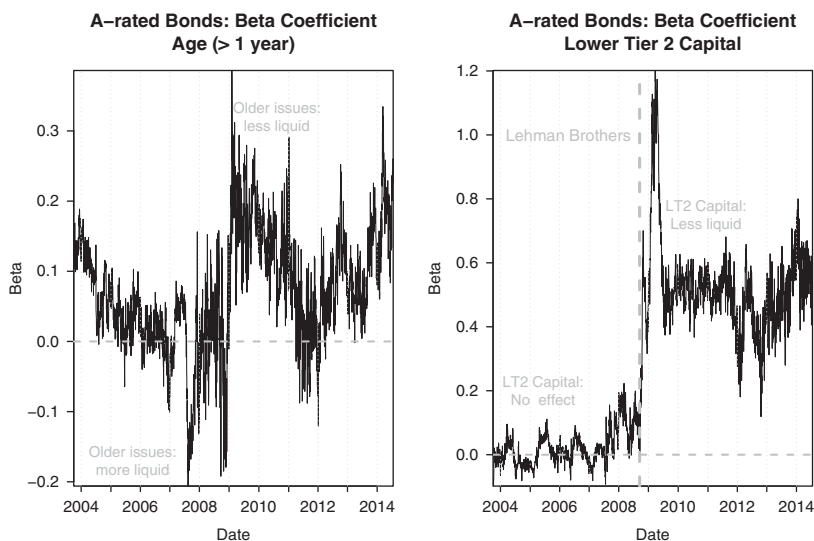


Figure 3. Beta parameters ( $\beta$ ) for log notional (left) and seniority indicator (right).

Table 1. Average of daily standard deviations by variable and rating class.

Rating	log (Duration)	log(Notional amount)	Coupon	Age	Financial	Seniority	Collateralised	Tier (lower tier 2 debt)
AAA	0.82	0.78	1.28	0.15	0.90	0.10	0.26	NA
AA	0.56	0.73	1.41	0.13	0.49	0.45	0.22	0.11
A	0.54	0.63	1.29	0.12	0.47	0.48	0.10	0.14
BBB	0.48	0.57	1.23	0.12	0.67	0.48	0.12	0.08

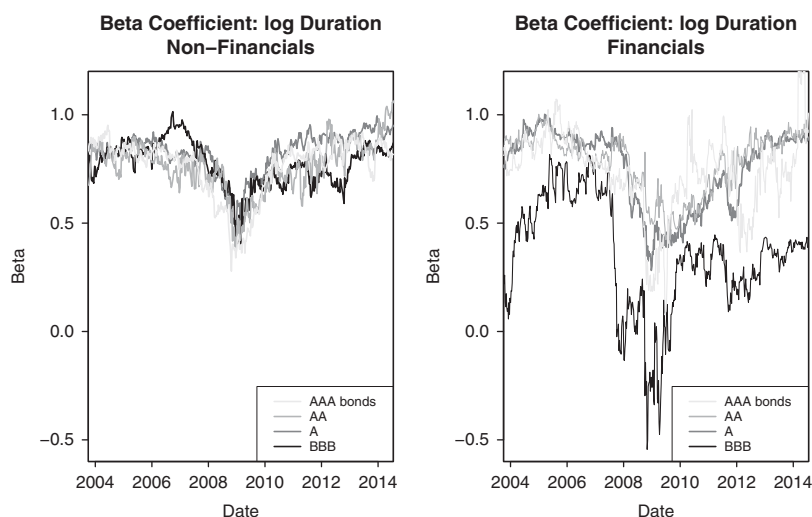


**Figure 4.** Beta parameters ( $\beta$ ) for the age indicator (left) and capital tier (lower tier (LT2)) indicator (right).

The duration  $\beta$  parameter for both financial issuers and non-financial issuers, for A-rated bonds can be seen in Figure 2. The duration coefficient,  $\beta_1$ , is close to 1 ( $\sim 0.9$ ) for both financial and non-financial issuers, before the crisis. In 2008, just after the nationalisation of Northern Rock, the coefficient dropped substantially. The duration  $\beta$  coefficient for non-financials recovered to pre-crisis levels far more quickly than its financials counterpart (mid-2010 versus beginning 2013).

Figure 3 shows the evolution of the  $\beta$ -coefficient for log notional amount (left) and for the senior/subordinate indicator (right). The negative coefficient of the notional amount  $\beta$  parameter (Figure 3, left) indicates that bigger issues generally have lower bid-ask spreads. This relationship broke down at the height of the crisis in 2009, suggesting that large issues were more difficult to trade at the desired volumes. The apparent unexpected result could be a data artefact; as large issues were the only bonds trading at the time, the quotes for small bonds were not updated. On the right (Figure 3), we can see that senior bonds did not trade at different levels of liquidity before the credit crunch. The onset of the credit crunch caused senior bonds to trade at much lower bid-ask spreads. The increased (decreased) liquidity of senior (subordinate) bonds seems to support the often-quoted “flight-to-quality” of safer senior (financial) bonds.

Lastly, Figure 4 displays the  $\beta$  parameters for the indicator variables related to bond age ( $>1$  year) (left) and capital tier (LT2) (right). The coefficient for the age indicator (Figure 4, left) is rather volatile and relatively small in magnitude; with a value of  $\sim 0.1$  on average, implying that recent issues (age  $< 1$  year) typically have a bid-ask spread that is 10% narrower than older issues (age  $> 1$  year). The sign of the  $\beta$  parameter is according to expectations on most days; older issues ( $> 1$  year) appear to be less liquid. However, from late 2007 to the end of 2008, the coefficient was negative indicating that newly issued bonds were more difficult to trade at that time, perhaps because short-term traders suddenly found it difficult to offload recently purchased new issues. A financial institution’s debt is capital that serves as protection of depositors from a regulatory viewpoint and the regulator categorises this capital in tiers. From a regulatory perspective of a bank’s capital, only



**Figure 5.** The weekly duration coefficient,  $\beta$ , for non-financial issuers (left) and financials (right) for all four rating categories.

25% of a bank's total capital can be LT2 debt and is generally the easiest and cheapest to issue. Not unsurprisingly we can also observe the “flight-to-quality/liquidity” in Figure 4 (right), where LT2 capital becomes more illiquid after the onset of the credit crunch, with extreme levels of illiquidity during early 2009.

Throughout the results section, the focus will be on displaying numerical results for the models fitted to A-rated bonds, as this corresponds most closely to the typical credit quality of an insurance portfolio. However, the modelling approach also allows for  $\beta$  coefficients to be compared across rating, providing insight into market behaviour at different segments of the credit quality spectrum. For example, the duration coefficients ( $\beta_1$ ) for financials and non-financials can be compared across rating.

The evolution of the duration parameter can be seen in Figure 5. The weekly duration coefficient for non-financial issuers (Figure 5, left) is both similar in magnitude across rating category and evolves similarly over time across category. The equivalent coefficient for financial issuers ( $\beta_{1,FIN}$  in equation (1)) follows a similar evolution over time (drops lower than non-financial parameter and recovers slower, seen in Figure 2) for AAA-, AA- and A-rated bonds. It is important to emphasise that the small sample size of AAA-rated financial issuers (bonds) post 2010 is responsible for the volatility in the AAA-rated coefficient; for example, on 15 September 2011 only 12 such securities are present in the data set. The BBB-rated coefficient ( $\beta_{1,FIN}$ ) is very different from the other rating categories, before, during and after the credit crunch.

## 5.2. Modelling credit spread

As remarked earlier, we use Markit's Annual Benchmark Spread as our measure of credit spread. Similar to the bid-ask spread model, we continue to review some of the model parameters ( $\gamma_k$ ) for several sub-models (by rating and date, as in equation (3)). We also refer back to Table 1 to aid in the interpretation, please note that the RBAS is modelled as  $\exp(\epsilon_{BAS})$ . Figure 6 shows the  $\gamma_k$  coefficients for two indicator variables; non-financial issuer (financial issuer) and senior

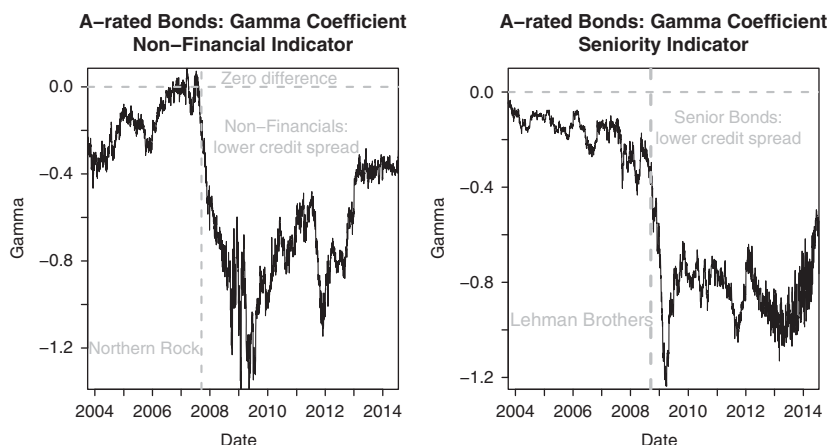


Figure 6.  $\gamma$  coefficient for non-financials (left) and senior (right) bond indicators.

(subordinate) issuer. We can reasonably expect bonds issued by financials and non-financials, *ceteris paribus*, to trade at similar prices before the crisis.

Given the relative instability of the financial services industry (particularly banks) during the crisis, we expect to see financials trade at lower prices/higher spreads (yields) after the Northern Rock bank run (14 September 2007). Figure 6 shows that yields of financial- and non-financial issuers, *ceteris paribus*, started to diverge at the time of the Northern Rock bank run and are yet to recover fully to pre-crisis levels. Similarly, senior and subordinate bonds traded at similar prices until mid-2007, but have since diverged. The negative coefficient for the relevant  $\gamma$  coefficient, implying lower yields for senior bonds, is not surprising, nor is the fact that the coefficient was close to zero; in a low default regime, recovery rates (affected by seniority status) are not likely to be an important determinant of bond prices. In a regime with high (perceived) default risk (premia), especially for financial issuers, which issue most subordinate bonds, recovery rates are more likely to impact an investor's decision.

Figure 7 shows the evolution of the  $\gamma$  parameters for duration and the relative liquidity proxy, RBAS. The  $\gamma$  coefficient of duration (NF) is positive for most days during the observed time period, indicating a rising credit spread curve. From close inspection of Figure 7 (left), we can also see that the duration parameter started its steep drop just days/weeks before the indicated Northern Rock Event (14 September 2007). Lastly, the zero/negative value of this parameter indicates a flat or falling credit spread curve; this could be interpreted as the market trading on price rather than yield, where short-term concerns over the value of investments dominate an investor's behaviour. Given the specification of equation (3), the  $\gamma$  parameter of RBAS (Figure 7, right) is directly related to the size of the liquidity premium we will extract in the next section. At this point it suffices to observe that the relative liquidity proxy is positively priced on all days during the sample period, except for very brief during 2006/2007.

Figure 8 shows the evolution of the  $\gamma$  coefficients for coupon (left) and the collateralised indicator (right). The  $\gamma$  parameter for Coupon rate is positive throughout the sample period, indicating that bonds with higher coupon rates trade at higher spreads. Please note coupon rates are expressed as whole numbers; for example the effect of a 5% paying bonds would be  $5 \times \gamma$ . This is also according

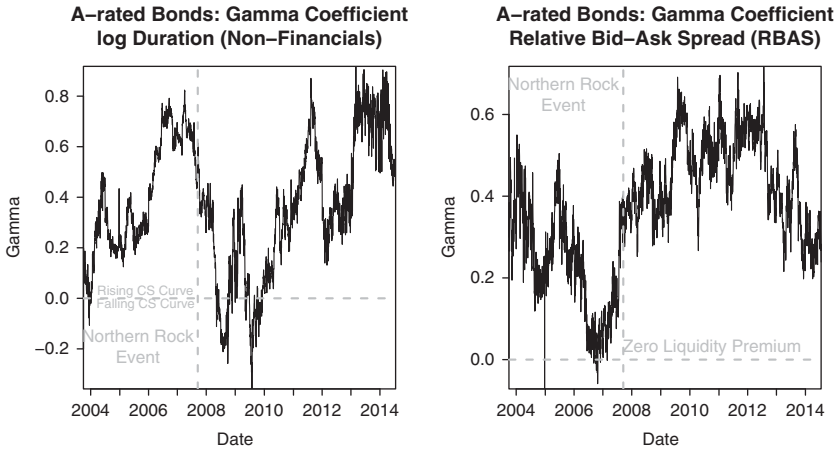


Figure 7.  $\gamma$  coefficient for log duration (non-financials) on the left and RBAS on the right.

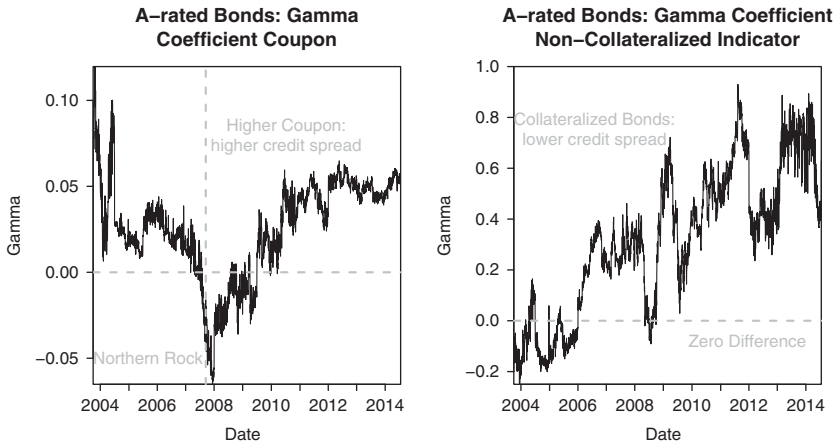
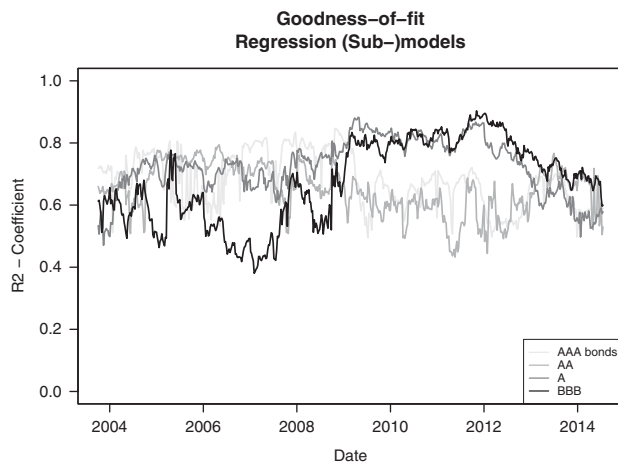


Figure 8.  $\gamma$  coefficient for coupon (left) and collateralised indicator (right).

to expectations and in line with literature (e.g. Leland, 1994) that finds a “tax effect”, where the underlying idea is that bonds with a low coupon rate have a more favourable tax treatment than high coupon paying bonds. We would also expect collateralised bonds to trade at lower credit spread, which is what we observe for most of the sample period (Figure 8, right). The zero/negative coefficient from 2004–2006 is unexpected at first sight but might be explained by the dynamics of supply and demand for, for example, mortgage-backed securities.

Finally, we consider how well the model (equation (3)) explains the variation in credit spreads. Figure 9 shows the aggregated weekly  $R^2$ -statistic over time for all four rating categories. The  $R^2$ -statistic is a commonly used indicator of goodness-of-fit in linear regression and is defined as the ratio of explained variance (variance of model’s predictions) to the total variance (sample variance of dependent variable). As can be seen, the model describes the data very well, but varies by both rating and time. Three additional observations can be made;  $R^2$  is high in general, there is no period for



**Figure 9.** Variation of the coefficient of determination,  $R^2$ , for the credit spreads model over time (weekly) and by rating category.

which  $R^2$  appears substantially lower and between 2009 and 2013 the  $R^2$  of A- and BBB-rated bonds seems to be substantially higher.

### 5.3. Liquidity premium estimates

As remarked earlier, we will investigate the liquidity premium estimates both in number of bps and as a proportion of total credit spread. Whereas this section will focus on the numerical results for A-rated bonds in particular, similar results for other rating categories can be found in Appendix. For A-rated bonds, Figure 10 provides us the following:

- (left) shows us a time-varying decomposition of the median credit spreads into a liquidity (black) and non-liquidity component (grey);
- (middle) shows us the liquidity component (in bps);
- (right) shows us the liquidity component (as proportion of spread).

As liquidity premia in Figure 10 (middle and right) clearly vary over time, we conclude that the liquidity premium of an A-rated bond of average liquidity is time dependent. The time dependency of liquidity premia is not limited to bps (if Figure 10, right, were constant, liquidity premia would simply move proportionally with credit spreads), but extends to the proportion of credit spreads. In the pre-crisis period, average liquidity premia appear low (relative to the rest of the sample period) and somewhat volatile. Just before the start of the credit crunch (2006–2007), average liquidity premia were near zero (Figure 10, right) on low credit spreads in general (Figure 10, left). The onset of the credit crunch caused the liquidity premium to rise from near-zero levels to ~50% of credit spreads.

The non-liquidity component, consisting of both the expected default losses and a premia, also increased (in bps) drastically. As expected default losses would only have increased marginally during this period, we can conclude that investors became much more risk averse during this time. Greater levels of uncertainty, or perceived uncertainty, in default estimates would have led to greater risk aversion, which is the reason a credit risk premium exists in the first place.

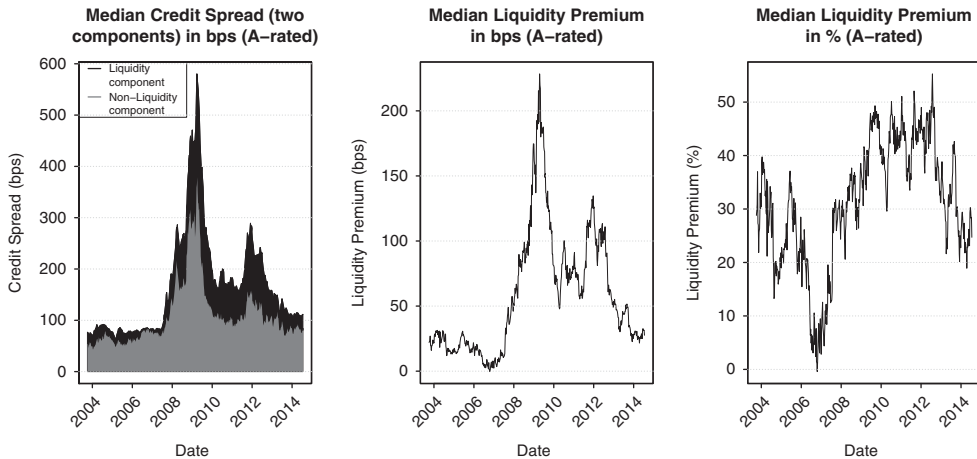


Figure 10. Decomposition of credit spread (left) for A-rated bonds of average liquidity into a liquidity and non-liquidity component; liquidity component of credit spread (middle) in basis points and the liquidity component of credit spread as a proportion of total credit spread (right).

The non-liquidity component, consisting of both the expected default losses and (credit) risk premia, also increased (in bps) dramatically. This increase would reflect a number of factors. First, within the economic cycle and in the context of the crisis, short-term expected default probabilities would have risen even if a bond's rating was unchanged. Arguably, though, this could only contribute in a small way to the overall increase. Second, investors' levels of risk aversion have increased significantly during the crisis, pushing up risk premia. Third, there might have been increased uncertainty in what future default probabilities and recovery rates would be. This additional parameter uncertainty attracts its own risk premium, which would, therefore, have risen during the crisis.

Liquidity premia (Figure 10, right) were relatively high and stable for several years during/after the credit crunch (Figure 10, right), irrespective of levels of credit spreads (Figure 10, left) and appear to have recently started to decline at the start of 2013.

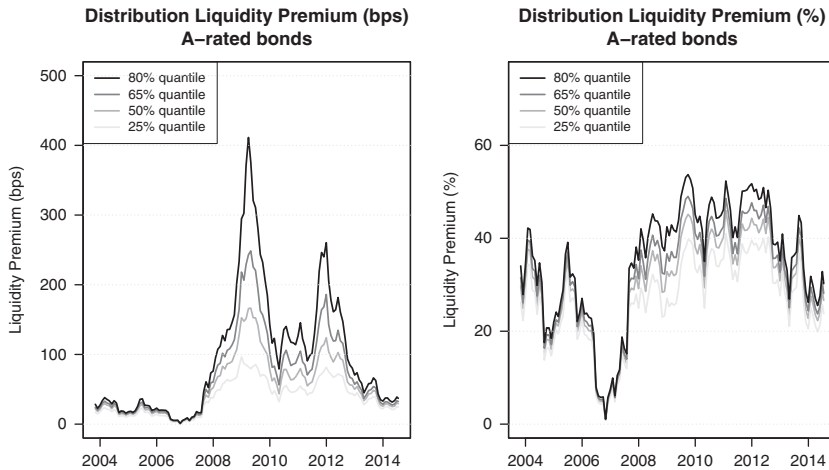
Rather than looking at the average (point-estimate) of the liquidity premia over time, Figure 11 investigates the distribution of liquidity premia by plotting various percentiles of the daily distributions (in bps) over time, on a monthly basis. In the left-hand plot, for example, an 80% quantile of 200 on a given date means that 20% of bonds had a liquidity premium of more than 200 bps on that date.

The distribution of liquidity premia is tight pre-crisis, but widens substantially during the 2008–2013 period, only recently becoming tighter again. The skew of the distribution (long upper tail) is a direct result of the skewed distribution of RBAS as  $\exp(\varepsilon_{BAS})$  from equation (1).

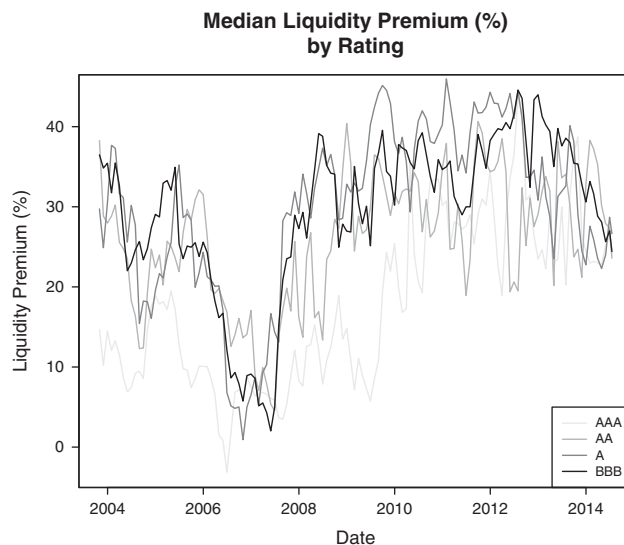
Lastly, we compare estimates of average liquidity premia across rating category in Figure 12.

Taking monthly estimates to remove most of the very short-term volatility of the time series to improve legibility, we can see that before the crisis, the four categories behaved similarly,



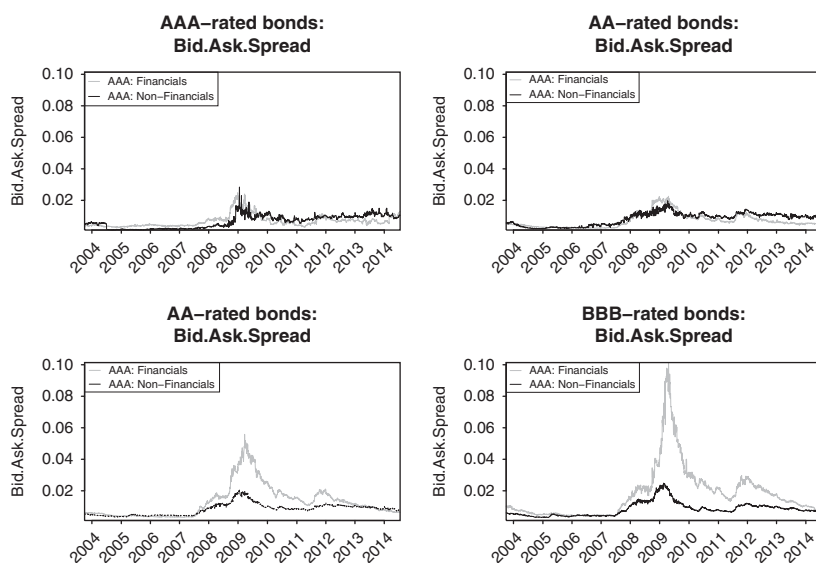


**Figure 11.** Time-varying nature of four weekly quantiles of daily liquidity premium distributions, in basis points (bps, left) and in proportion of spread (right).



**Figure 12.** Monthly estimates of median liquidity premia across rating category.

with the exception of the AAA-rated bonds, which saw far smaller liquidity premia. All rating categories display very low premia (0%–10%) from mid-2006 to mid-2007 and shoot up as a response to the credit crunch (again, AAA-rated bonds are the exception). After the start of the credit crunch, the A- and BBB-rated bonds appear to behave differently; whereas AAA- and AA-rated bonds return to pre-crisis levels (AAA-rated slightly elevated), bonds of lower credit ratings see far higher liquidity premia for a prolonged period of time, starting to return to pre-crisis levels in 2013. In general, bonds with a lower credit rating have higher liquidity premia (as proportion of spread).



**Figure 13.** Shown on the same scale, the bid-ask spread for all IG ratings, both financial and non-financial issuers, increased dramatically during the financial crisis. Note that the abnormality for AAA-rated non-financials is owing to a methodology change in the data; several non-financial issuers became financial issuers on that day.

## 6. Additional Analyses

We perform two additional analyses; the first is related to the choice and derivation of the new, relative liquidity proxy. The second takes a closer look at some of the properties of the RBAS that might make it particularly suitable for ALM modelling purposes.

### 6.1. Alternative modelling approach

The RBAS is, to our knowledge, the only truly relative liquidity proxy, which exhibits the perhaps counterintuitive property of having a constant average value, on a daily basis. As RBAS is defined as the exponential of the residual term from equation (1), its distribution, on a daily basis, is always centred around 1 (standard log-normal for normally distributed residuals), irrespective of economic climate. The distribution for RBAS can either be wider or narrower, depending on the daily fit of the regression model (equation (1)). Its relative nature and design does bring the attractive property of being uncorrelated with common bond characteristics. Using the same period and bond universe, a similar set of regression models are specified, but with the “raw” bid-ask spread as liquidity proxy. The methodology to extract the liquidity premia is different and does not create a hypothetical perfectly liquid alternative; the method of premium extraction is based on Dick-Nielsen *et al.* (2012).

The use of bid-ask spreads, or indirect measures of the bid-ask spread such as the roll-measure (as recently in Bao *et al.*, 2011) or Imputed Roundtrip Costs (Feldhütter, 2012), have frequently been used to study the effect of illiquidity on asset prices. Figure 13 clearly shows that the time series of daily median bid-ask spread for investment grade bonds is highly time-, rating- and financial/non-financial dependent, with the spread on financials increasing by much more during the crisis.

Again, we formulate regression models of credit spreads and bond characteristics (identical covariates to equation (3)), now including the bid-ask spread directly instead of RBAS:

$$\begin{aligned}
 (CS(i, r, t)) = & c(r, t) \\
 & + \theta_{1, \text{FIN}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{FIN}}(i) \\
 & + \theta_{1, \text{NF}}(r, t) \times \log \text{Duration}(i, t) \times I_{\text{NF}}(i) \\
 & + \theta_2(r, t) \times \log \text{Notional}(i, t) \\
 & + \theta_3(r, t) \times \text{Coupon}(i, t) \\
 & + \theta_4(r, t) \times \text{BAS}(i, t) \\
 & + \sum_k \theta_k(r, t) \times I_k(i, t) \\
 & + \epsilon_{\text{CS}}(i, t) \quad (\text{residual})
 \end{aligned} \tag{6}$$

As we are modelling credit spread rather than  $\log(\text{CS})$ , we can work with coefficients and covariates directly to see their contribution to spread in bps. Instead of estimating the perfectly liquid equivalent bond, we follow an estimation procedure similar to Dick-Nielsen *et al.* (2012). The liquidity score for each bond is defined as  $\theta_4(r, t) \times \text{bid-ask spread}(i, t)$ . Within each rating category (AAA, AA, A, BBB) and day, we sort all bonds on their liquidity score. Then, the size of the illiquidity contribution to the spread, for an average bond, is defined as the 50% quantile minus the 5% quantile of the liquidity score distribution in a particular bucket ( $\theta \times (\text{BAS}_{50} - \text{BAS}_5)$ ). Therefore, the liquidity contribution measures the difference in credit spread between a bond of average liquidity and a bond that is very liquid. Compared with our approach of estimating perfectly liquid bonds, this measure is relative; the 5% quantile represents a very liquid bond on a particular day.

Comparing the time series of daily liquidity premium estimates for the A-rated bucket, with our estimates of median A-rated liquidity premia shows (Figure 14) that the two move together, but are quite different. The alternative approach shows spreads of near zero for the entire pre-crisis period (bid-ask spread is not significantly priced), seems to react to the credit crunch more slowly, peaks around similar levels but drops much further in 2010–2011, increases drastically for only a few weeks during the European debt crisis from ~15% of spread to 65% of spread, only to return to near-zero levels very quickly.

## 6.2. Investigating RBAS properties

For potential ALM purposes, we are ultimately interested in the RBAS of individual bonds. As remarked earlier, the liquidity proxy is entirely relative (daily distribution centred around 1) and uncorrelated with common bond characteristics, all of which allows for the direct comparison of intrinsic bond liquidity. To gain a better insight into the properties of our relative liquidity measure, we have taken a set of financial and non-financial issuers with multiple bonds outstanding on a particular day and graphically explore whether there seems to be evidence for an issuer-specific liquidity effect. It is noteworthy that issuer-specific liquidity has, to our knowledge, only been briefly explored by Dick-Nielsen *et al.* (2012), who considered an issuer-specific liquidity proxy (non-zero-trading-days for issuer). Including additional covariates in our model such as number of bonds outstanding or total notional outstanding yields a  $\beta$  parameter that is largely insignificant and has been omitted from equation (1). In Figures 15 and 16 we show the bid-ask spread (left) and RBAS (right) for financial and non-financial issuers, respectively.

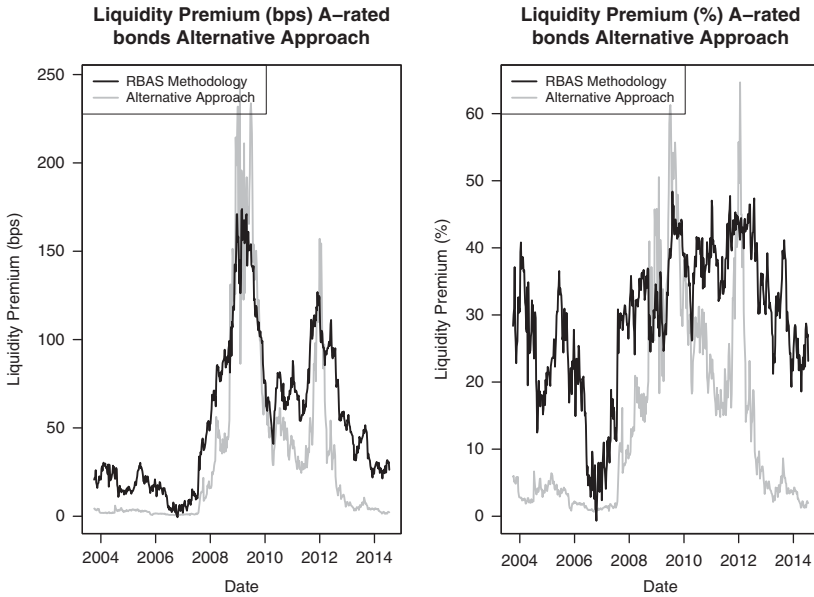


Figure 14. Comparison of A-rated, weekly aggregated, liquidity estimates in basis points (bps, left) and per cent of spread (right) for our modelling approach (black) and the alternative approach (grey).

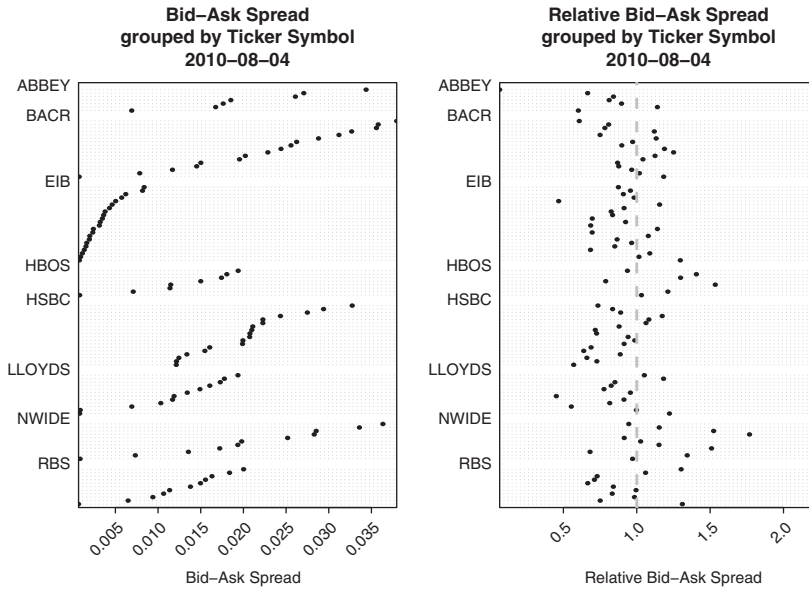
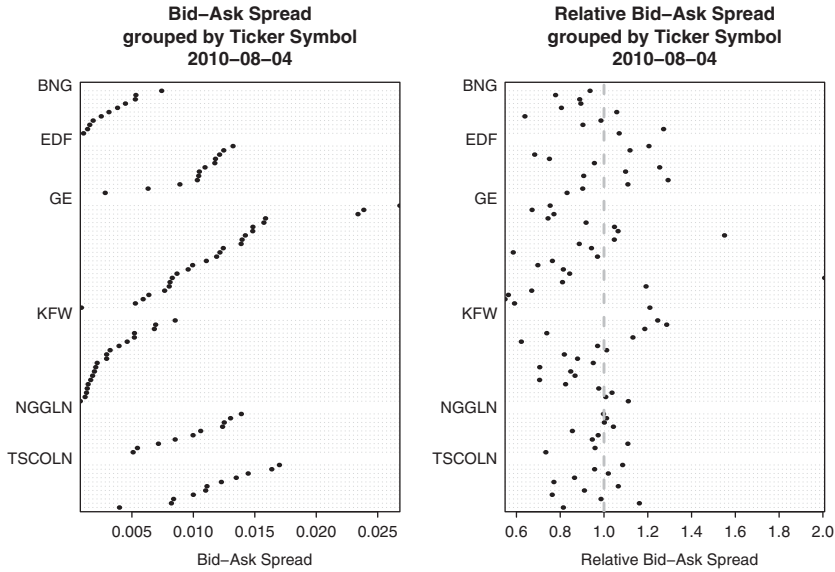


Figure 15. Bid-ask spreads (left) and relative bid-ask spread (RBAS) (right) for bonds issued by selected financials. Ordering of bonds (by issuer and then by magnitude of BAS) on the left is preserved in the right-hand plot.

Two observations are important to make; first, daily BAS and RBAS for individual bonds/issuers are uncorrelated. The second observation is related to the issuer-specific liquidity. Whereas we have omitted the issuer-specific variables from the model, Figure 15 appears to display some issuer-specific



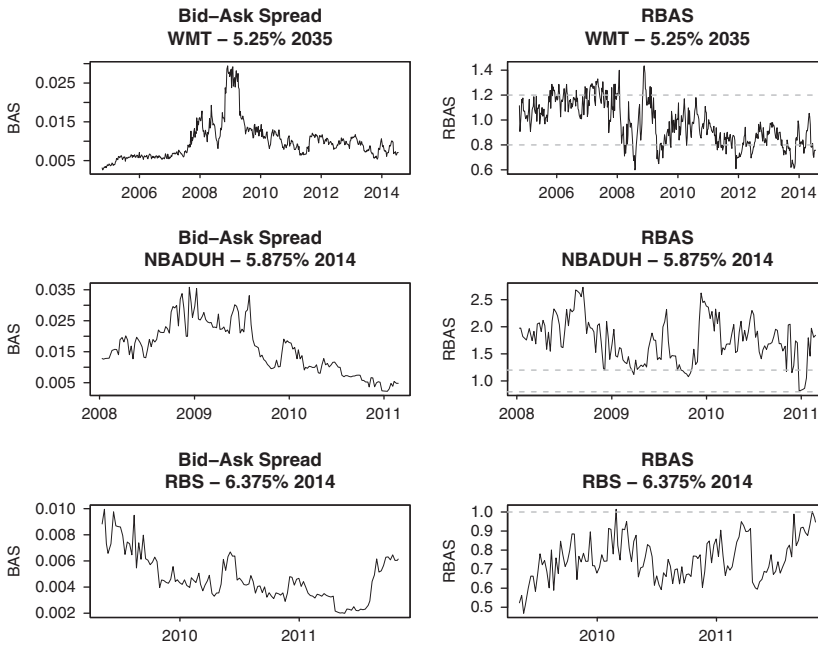
**Figure 16.** Bid-ask spreads (left) and relative bid-ask spread (RBAS) (right) for bonds issued by selected non-financials. Ordering of bonds (by issuer and then by magnitude of BAS) on the left is preserved in the right-hand plot.

effect for financials *during* the credit crisis. We define issuer-specific liquidity as “generally more or less liquid than average”, where in Figure 15 we can see that issuers ABBEY, HSBC and LLOYDS seem to have most bonds outstanding with  $RBAS > 1$ . This effect is very limited for non-financial issuers (Figure 16), where perhaps the same issuer-specific liquidity effect can be observed for GE (General Electric).

The evolution of a bond’s relative liquidity over time is important to consider with respect to potential ALM modelling. Estimates for RBAS are relatively robust over time, meaning that over short–medium periods of time, RBAS changes little. The volatility of RBAS is dependent on volatility of model parameters  $\beta_k$  in equation 1, and dependent on the movement of the bond’s bid-ask spread; both move in response to a changing market (model parameters) and idiosyncratic shocks. Figure 17 shows the evolution of weekly bid-ask spreads (left) and RBAS (right) of three bonds over a long period of time (multiple years) and it is clear that these bonds, despite short-term volatility, operate at three different points of the RBAS spectrum. Please note the particular issue of the Royal Bank of Scotland (Figure 17, third row), which in recent years appears to be consistently more liquid compared with identical bonds; this is likely the result of the government backing of RBS.

## 7. Conclusion and Discussion

In this paper we have taken a new reduced-form modelling approach to estimating liquidity premia on corporate bonds that has few data constraints compared with previous methodologies using CDS data, structural models or reduced-form models relying on external data for credit risk control variables. We show the time-varying nature of liquidity premia for various rating categories over an 11-year period capturing a benign financial climate, the financial crisis and more recent years.



**Figure 17.** Bid-ask spreads (left) and relative bid-ask spread (RBAS) (right) for three individual bonds over time.

We observe liquidity premia, as a proportion of total credit spread, to be bigger for bonds with lower credit ratings and emphasise the distribution of liquidity premia instead of point-estimates only. The sign, magnitude and evolution of model parameters provides insight into market dynamics, especially how relationships changed, broke down or remained stable during the credit crunch. The evolution of parameter estimates in more recent years indicates how market dynamics have recovered, not yet recovered or changed as a result of the crisis.

Despite the economically intuitive and significant changes of model parameters and outcomes over time, the modelling approach in this paper has no explicit time component. One could apply time series analysis to the daily changes in values for RBAS (on the individual bond level) and its coefficient (for both the individual and aggregate levels). This could provide forward-looking estimates of aggregate or bond-specific liquidity premia with relatively high frequency (daily/weekly/monthly).

This paper introduces a new modelling approach to extracting liquidity premia and discusses its most notable features, but the model specification does allow for more complex forms to be introduced. Perhaps, most importantly, a liquidity premium-term structure is not explored.

Another area of future work, of particular interest to buy-and-hold investors and to Solvency II regulators, may focus on the extent investors “earn” the liquidity premium as a function of expected holding period. All estimates of (il)liquidity premia (in bps) relate to the additional expected return when holding the bond to maturity. Even buy-and-hold investors that have the intention to do so will be faced with an expected holding period that is shorter than maturity because of, for instance, a mandate to sell bonds below BBB.

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### Appendix: A Numerical Results Liquidity Premia

The numerical results with respect to liquidity premia for A-rated bonds (Figure 10) are replicated here for the three other rating categories.

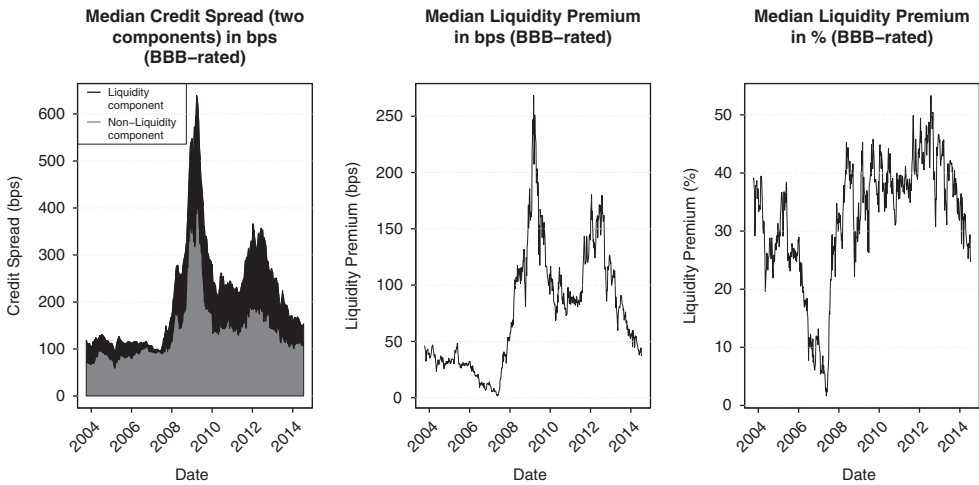


Figure A.1. Decomposition of credit spread (left) for BBB-rated bonds of average liquidity, into a liquidity and non-liquidity component; liquidity component of credit spread (middle) in basis points and the liquidity component of credit spread as a proportion of total credit spread (right).

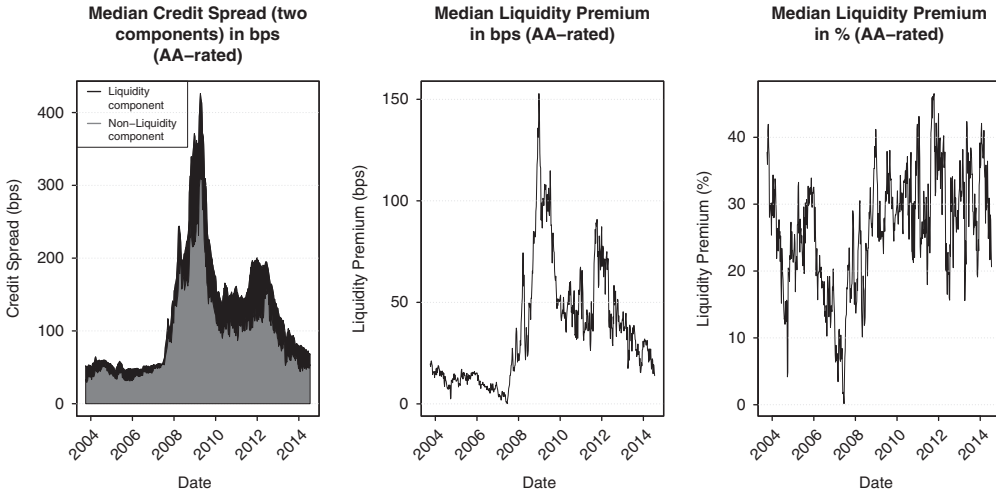


Figure A.2. Decomposition of credit spread (left) for AA-rated bonds of average liquidity, into a liquidity and non-liquidity component; liquidity component of credit spread (middle) in basis points and the liquidity component of credit spread as a proportion of total credit spread (right).

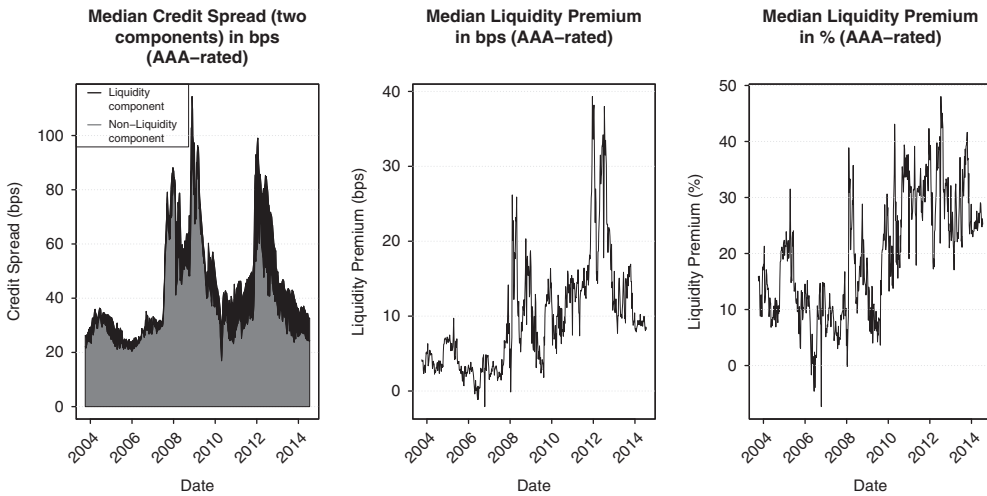


Figure A.3. Decomposition of credit spread (left) for AAA-rated bonds of average liquidity, into a liquidity and non-liquidity component; liquidity component of credit spread (middle) in basis points and the liquidity component of credit spread as a proportion of total credit spread (right).

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